# Comparison of Different Offset Arrangements of Roller Followers of Constant-Diameter Cam Mechanisms

Yen-Nien Chen and Kuan-Lun Hsu*

*Abstract*— **This paper first presents analytical expressions for the cam profile determination of constant-diameter cam mechanisms with different offset arrangements of roller followers. Afterwards, force analyses are performed for each offset arrangements of roller followers. Based on the obtained results a criteria for a preferable offset arrangement for the constant diameter cam mechanism is addressed in this paper.**

## I. Introduction

Cam mechanisms have been commonly used in the modern industry applications, such as automatic tool changers (ATC), internal combustion engines, and pick-and-place devices [1], because an ordinary motion (either rotation or translation) can be transformed into intricate output motions efficiently. However, the common-used type, the force closed type [2], cam mechanisms usually equip with the return spring used to keep the follower in contact with the cam. While a larger inertia force is exerted on the follower, the loss of contact between the cam and the follower still may occur due to the insufficiency of the retaining spring force, which causes an undesired phenomenon called the follower jump [2]. The fluctuating jump may lead to serious damages of the cam surfaces result in early failure of the cam mechanism. To avoid the follower jump, the form closed cam mechanisms are more preferable. The advantage of a form-closed cam mechanism is that the cam can create multiple point contact with the follower without using return spring. For a form-closed cam mechanism, a constant-diameter cam mechanism with an offset roller follower has been discussed for the effect of reducing pressure angle [3]. However, fewer studies have been conducted on comparing the different offset arrangements of the roller followers. Therefore, this paper attempts to compare four offset arrangements of the roller followers, and suggest a best type of the offset roller follower of the constant-diameter cam mechanism.

## II. Cam Contour of Constant-Diameter Cam Mechanism with an Offset Roller Follower

Fig.1 shows the four types of constant-diameter cam mechanisms with an offset roller follower. Link 1, 2 and 3 are the ground link, the cam and the follower, respectively. $I_{12}$, $I_{23}$, $I_{13}$ are the instant centers of the kinematic pairs between the links. Setting a Cartesian coordinate embedded to the cam with its origin at fixed pivot O. The angular displacement and angular velocity of the cam is denoted as θ and ω, respectively. The pressure angle of the top and bottom rollers can be denoted as $\phi_A$ and $\phi_B$. From the geometry of $\triangle C_aQO$ and $\triangle C_bQO$, $\phi_A$ and $\phi_B$ can be expressed as :

$$\phi_A = \tan^{-1}((q + e) / L(\theta)) \quad (1)$$

$$\phi_B = \tan^{-1}((q + e) /(d - L(\theta)) \quad (2)$$

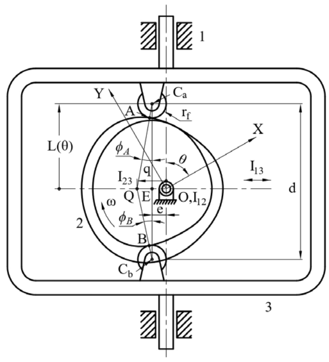
where L(θ) is the displacement function of the follower

$$L(\theta) = r_b + r_f + S(\theta) \quad (3)$$

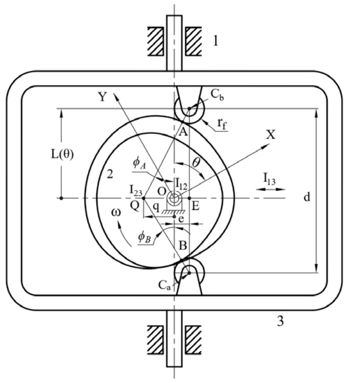
where $r_b$ and $r_f$ are the radii of the base circle and the roller, respectively. S(θ) is the follower motion program. Besides, offset e is positive when the roller is on the right, and e is negative when the roller is on the left. Therefore, by applying the concept of velocity instant center [4], the parametric equations of the cam profile can be determined as :

$$\boldsymbol{O_2A} = \boldsymbol{O_2E} + \boldsymbol{EC_a} + \boldsymbol{C_aA}$$
$$= e\angle(\theta+\pi/2)+L(\theta)\angle(\theta)+rf\angle(\theta + \pi - \phi_A) \quad (4)$$

$$\boldsymbol{O_2B} = \boldsymbol{O_2E} + \boldsymbol{EC_b} + \boldsymbol{C_bB}$$
$$= e\angle(\theta + \pi/2) + (d - L(\theta))\angle(\theta + \pi) + rf\angle(\theta + \phi_B) \quad (5)$$



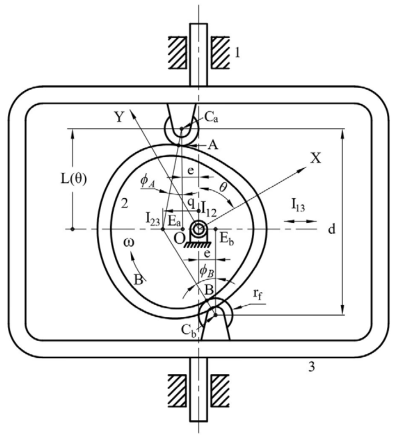

(a) both rollers on the left



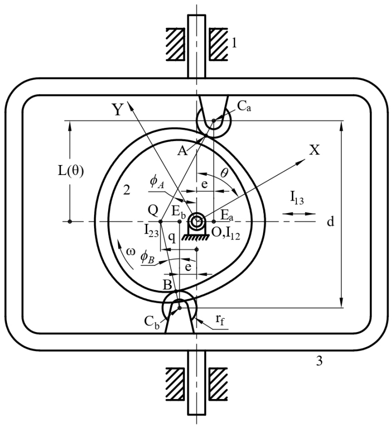

(b) both rollers on the right

Y. N. Chen currently is a graduate student under the advisory of Prof. Kuan-Lun Hsu in Department of Mechanical Engineering at National Taiwan University, Taipei 10607, Taiwan and expect to graduate by June 2022 (e-mail: r09522609@ntu.edu.tw).

K. L. Hsu* is with the Mechanical Engineering Department, National Taiwan University, Taipei, 10617 Taiwan (e-mail: kuanlunhsu@ntu.edu.tw).




(c) top roller on the left and lower roller on the right

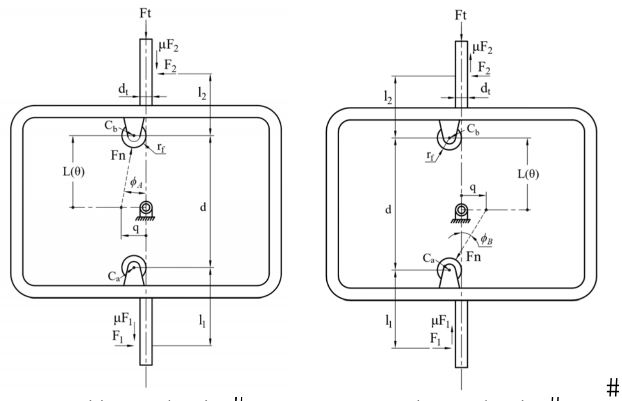



(d) top roller on the right and lower roller on the left

Figure 1. four types of constant-diameter cam mechanisms with offset roller followers

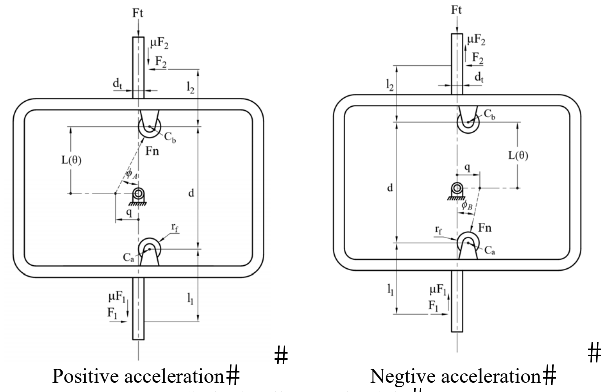
## III. Force analysis of constant-diameter cam mechanisms with an offset roller follower

To investigate the performance of the force transmissions of the cam mechanisms with different offset arrangements, a force analysis is provided here. Assume that only the upper roller takes the contact force when the acceleration of the follower is upward; and only the lower roller takes the contact force when the acceleration of the follower is downward. Thus, the free body diagram of the follower can be drawn as Fig.2.



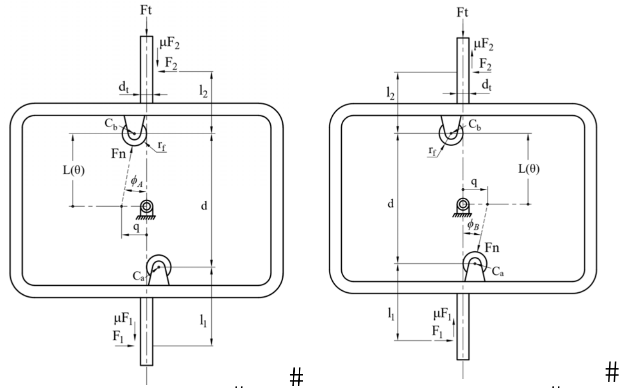

Positive acceleration# Negtive acceleration#

(a) both rollers on the left



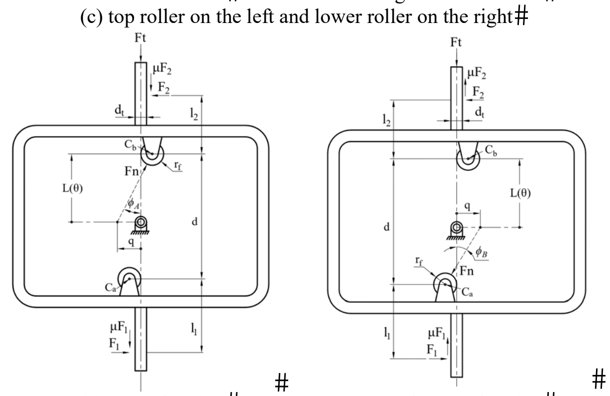

Positive acceleration# Negtive acceleration#

(b) both rollers on the right#




Positive acceleration# Negtive acceleration#

(c) top roller on the left and lower roller on the right#




Positive acceleration# Negtive acceleration#

(d) top roller on the right and lower roller on the left

Figure 2. Free body diagram

Based on the free body diagram shown in Fig.2, the equations of the force equilibrium and the moment equilibrium for the upward acceleration of the follower can be written as:

$$F_1 - F_2 + F_n \sin(\phi_A) = 0 \quad (6)$$

$$-\mu F_1 - \mu F_2 + F_n \cos(\phi_A) = F_t \quad (7)$$

$$F_1(l_1 + d + \mu d_t - e) + F_2(l_2 - \mu d_t/2 - e) = 0 \quad (8)$$

where $F_t$ is the sum of the inertia force and the gravity force, which can be denoted as :

$$F_t = M(g + A(\theta)) \quad (9)$$

where g is the gravity and A(θ) is the follower acceleration program. Equation (6) to Equation (8) can be expressed in a matrix form and solved simultaneously by an inverse matrix as denoted by Equation (10).

$$\begin{bmatrix} 1 & -1 & \sin(\phi_A) \\ -\mu & -\mu & \cos(\phi_A) \\ l_1 + d + \frac{d_t}{2} - e & l_2 - \frac{d_t}{2} - e & 0 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_n \end{bmatrix} = \begin{bmatrix} 0 \\ F_t \\ 0 \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_n \end{bmatrix} = \begin{bmatrix} 1 & -1 & \sin(\phi_A) \\ -\mu & -\mu & \cos(\phi_A) \\ l_1 + d + \frac{d_t}{2} - e & l_2 - \frac{d_t}{2} - e & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ F_t \\ 0 \end{bmatrix} \quad (10)$$

Similarly, in reference to the free body diagram in Fig.2, the equations of the force equilibrium and the moment equilibrium for the downward acceleration of the follower can be written as:

$$F_1 - F_2 + F_n\sin(\phi_B) = 0 \quad (11)$$

$$\mu F_1 + \mu F_2 + F_n\cos(\phi_B) = F_t \quad (12)$$

$$F_1(l_1+d+\mu d_t - e) + F_2(l_2 - \mu d_t/2 - e) = 0 \quad (13)$$

Equation (11) to Equation (13) can also be expressed in a matrix form and solved simultaneously by an inverse matrix as denoted by Equation (15).

$$\begin{bmatrix} 1 & -1 & -\sin(\phi_B) \\ \mu & \mu & -\cos(\phi_B) \\ l_1 - \frac{d_t}{2} + e & l_2 + d - \frac{d_t}{2} - e & 0 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_n \end{bmatrix} = \begin{bmatrix} 0 \\ F_t \\ 0 \end{bmatrix} \quad (14)$$

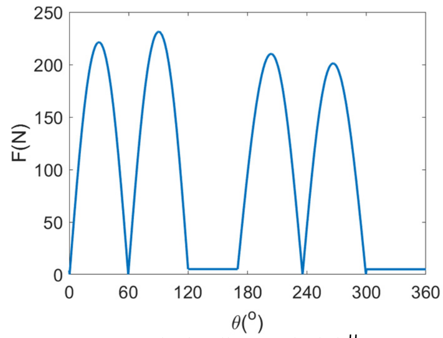
$$\begin{bmatrix} F_1 \\ F_2 \\ F_n \end{bmatrix} = \begin{bmatrix} 1 & -1 & -\sin(\phi_B) \\ \mu & \mu & -\cos(\phi_B) \\ l_1 - \frac{d_t}{2} + e & l_2 + d - \frac{d_t}{2} - e & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ F_t \\ 0 \end{bmatrix} \quad (15)$$

Based on Equation (10) and Equation (15), contact force $F_n$ can be obtained.

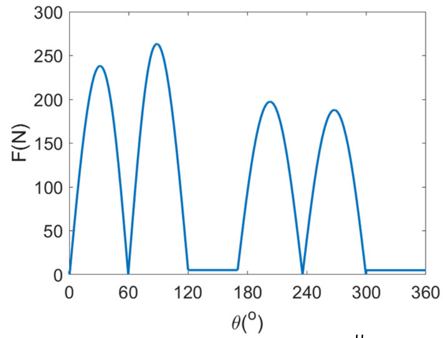
## IV. NUMERICAL EXAMPLES

Assume that the angular velocity of the cam is 1200 rpm. The follower is to rise 20 mm with a cycloidal motion while the cam rotates from 0° to 120°, dwell for the next 50°, return with a cycloidal motion for 130° cam rotation and dwell for the remaining 60°. The distance between two rollers d is 110 mm. The width of the follower $d_t$ is 10 mm. The offset of roller e is 10 mm. The initial value of $l_1$ and $l_2$ are 40 mm, respectively. The friction coefficient μ = 0.15 (unitless) and the follower mass M = 0.5 kg.

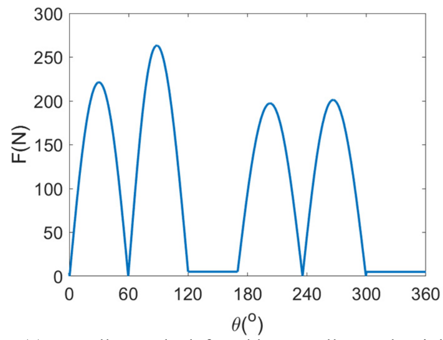
Contact force $F_n$ with respect to cam angle θ of the four types of the follower is shown in Fig.3, and their extreme values are correspondingly listed in Table.1. During the rising period, the follower with both rollers on the left has the smallest maximum $F_n$. On the other hand, the follower with both rollers on the right has the smallest maximum $F_n$ during the returning period. A cam mechanism with lower $F_n$ has a better performance of the force transmissions. Thus, the obtained result indicates that both rollers on the left side is preferred if a better force transmission is required during the rising period. On the other hand, both rollers on the right side is preferred if a better force transmission is required during the returning period.



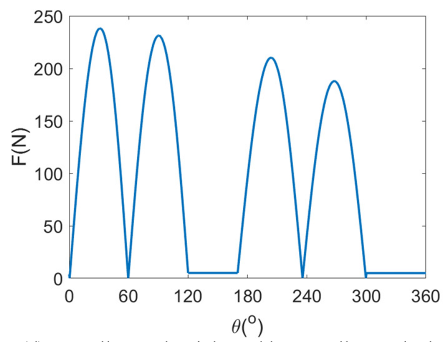

(a) both rollers on the left#




(b) both rollers on the right#




(c) top roller on the left and lower roller on the right#




(d) top roller on the right and lower roller on the left

Figure 3. Contact force Fn

TABLE 1 EXTREME VALUES OF $F_N$ IN FOUR TYPES OF CAM MECHANISMS.

| **Max $F_n$ (N)** | | **Type of follower** | | | |
|---|---|---|---|---|---|
| | | Fig. 2(a) | Fig. 2(b) | Fig. 2(c) | Fig. 2(d) |
| Rise | Degree(°) | 31.1 | 88.5 | 88.5 | 31.1 |
| | $F_n$(N) | 231.30 | 263.31 | 263.30 | 238.16 |
| Return | Degree(°) | 203.7 | 202.7 | 266.4 | 203.7 |
| | $F_n$(N) | 210.19 | 197.33 | 201.14 | 210.19 |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