An Integrated Design Process Combining Yan's Design Methodology with Geometric Constraint Programming - A Case Study of Ackerman Steering Mechanisms

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***Abstract*— This paper presents a design process which combines the creative design methodology proposed by Yan and geometric constraint programming (GCP) proposed by Kinzel, Schmiedeler, and Pennock. By merging these two methods together, a new-type mechanism can be designed systematically and effectively. An Ackerman steering mechanism is used as an illustrative example.**

I. Introduction

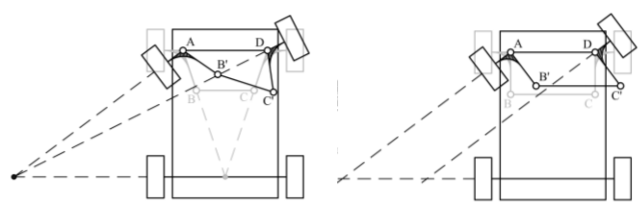
In early years, mechanisms designs usually depend on the designer's experience, creation and intuition. Therefore, the design process is not systematical until an effective method was proposed by Yan [1, 2]. His method can be used to generate all possible design concepts of movable devices with given topological and motion characteristics. The flexibility of his method has been verified successfully in many cases to develop new design concepts, such as a wheel damping mechanism [3], an automotive gear differentia [4], machining centers [5], flapping wing mechanism [6], and magnetic gears [7]. These intriguing works have proven the flexibility of Yan's design methodology on conceptualizing new designs. Nevertheless, there has been few studies continuing on developing the specifications of these derived concepts.

In order to realistically implement the derived concept, a dimensional synthesis must follow up to define the design specifications. Traditionally, the dimensional synthesis of planar linkages can be done graphically or analytically. Recently, a computer-oriented method named geometric constraint programming (GCP) [8, 9] is proposed by Kinzel, Schmiedeler, and Pennock to cope with the synthesis problem digitally. The presented method has several advantages over the graphical and the analytical method. It can easily be incorporated into a CAD software, which provides an user-friendly interface that allows the designer to visualize the design results directly. Besides, the mathematical constraints governing mechanism synthesis can be graphically imposed by adding constraints to sketch entities representing the mechanism geometry. Most important of all, the synthesized results with high accuracy will be provided immediately once all geometrical and dimensional constraints are applied correspondingly. For example, a meso-gripper was designed using the GCP technique [10]. The GCP technique could be advantageous in specifying the dimensions of synthesized mechanisms once the mechanism topologies are conceptualized ahead.

Given that a complete mechanism design ought to include both design concept generations and detailed engineering refinements, Yan's design methodology and geometric constraint programming are preferably integrated into a comprehensive procedure, which will be illustrated in this paper. An Ackerman steering mechanism [11] is used as a source device that engineers other feasible design concepts by creative mechanism design methodology [12-15]. Next, the geometric constraint method is utilized to facilitate the dimensional synthesis after which the new topology of the synthesized mechanism is developed. Besides, the prototyping of the presented design is implemented in a self-driven vehicle in A4 size.

II. Ackerman Steering Geometry

In Automobiles, Ackermann geometry [11] has been widely adopted as a criterion for the wheel steering mechanism for a smoother turning path of the vehicle. The main concept is to differ the turning angle of the front wheels so as to let their normal lines and the rear axle coincide at a point, the steering center, like Fig. 1(a) shown. In addition, while front wheels point straight ahead, the intersection of lines AB and CD is preferable on the middle point of the rear axle. This geometry could let both front wheels maintain pure rolling while turning, because they have the same rolling center. In contract, as shown in Fig. 1(b), the steering mechanism without satisfying Ackerman geometry would cause the vehicle slipping due to the traveling difference between inner and outer front-wheel.



(a) Ackerman geometry (b) Parallel four-bar linkage

Figure 1. Steering mechanism using four-bar linkage

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The simplest mechanism meeting Ackermann geometry is a four-bar linkage as shown in Fig. 1. However, this elementary configuration cannot satisfy Ackermann geometry with the change of the turning angle. To reduce the steering deviations, an alternate linkage capable of meeting more precision points is a superior option [12-15]. Thus, Yan's design methodology is applied for generating possible configurations to meet this need.

III. KINEMATIC CHAIN TOPOLOGY

Yan's creative design methodology can be applied for a new-type steering mechanism. Fig. 2 shows the procedure of Yan's creative design methodology.

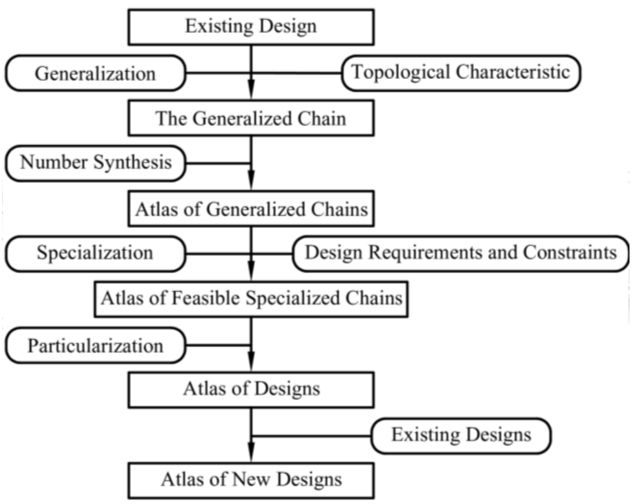
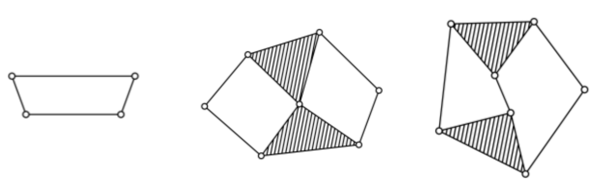


Figure 2. Yan's creative design methodology

Usually, Ackerman geometry is satisfied by adopting a four-bar linkage in Fig. 1(a), which can be generalized as the kinematic chain shown in Fig. 3(a). However, as mentioned in section II, a four-bar Ackerman mechanism can generate the steering precisely at three positions only. Since the Ackermann type steering mechanism reveals progressive deviations from ideal steering with increasing ranges of motion, we attempt to derive kinematic chains with more links but the same degrees of freedom (DOF). The generalized kinematic chains with six links and seven joints are applied, which would give five precision points to steering [12, 14]. The atlas of generalized kinematic chains with six links and seven joints are listed in Fig. 3(b) and (c), which are known as Watt chain and Stephenson chain, respectively.



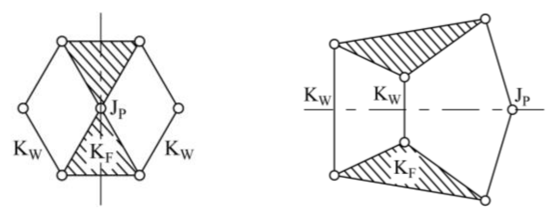
(a) four-bar chain (b) Watt chain (c) Stephenson chain

Figure 3. Atlas of generalized chains

Next, we are going to specialize those kinematic chains by the design requirement. The basic requirements regarding the topological characteristics of this integrated mechanism are concluded as follows:

1. It is a mechanism with 1 DOF.
2. It is a planar 6-link, 7-joint mechanism.
3. The mechanism is symmetric.
4. It has one ground link (K_F) and five kinematic links including two links connecting with wheels (K_W).
5. The ground link (K_F) must be a ternary link.
6. There are two wheel links (K_W) which are binary links.
7. There are six revolute joints and one prismatic joint (J_P).
8. The wheel link (K_W) and ground link (K_F) is connected with revolute joint.

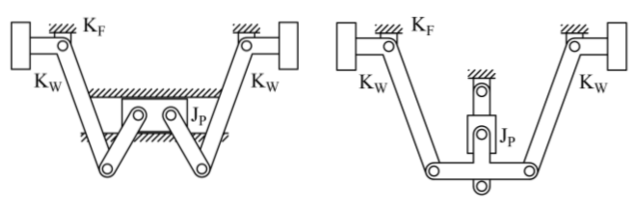
According to the symmetric condition, the prismatic joint (J_P) can be assigned on the symmetry axis. The feasible specialized chains are shown in Fig. 4. All of the unlabeled links and joints serve as the connecting links and revolute joints, respectively. Fig. 4(a) is the only one result derived from Watt kinematic chain while Fig. 4(b) is the only one result derived from Stephenson kinematic chain.



(a) (b)

Figure 4. Atlas of all feasible specialized chains

After having the atlas of specialized chains, the next step is to particularize the specialized chains in Fig. 4. Fig. 5(a) is the structural sketch of Fig. 4(a) through particularization, while Fig. 5(b) is corresponding to Fig. 4(b).



(a) (b)

Figure 5. Atlas of functional schematics of feasible specialized chains

Considering that our main goal is to design steering mechanism with more precision points, Fig. 5(b) cannot be applicable since it essentially is a four-bar Ackerman steering. Therefore, the Fig. 5(a) would be the specialized kinematic chain being discussed in the following sections.

IV. DIMENSION SYNTHESIS

Once the new configuration of Ackerman steering mechanism has been designed, the next step is to determine its dimensional parameters. There are many kinds of approaches [8, 9, 16-22] used for the dimensional synthesis of mechanical devices. A geometric constraint programming (GCP) technique is adopted in this research due to its favorable features aforementioned in section I Introduction. Geometric constraint programming can be carried out by any parametric CAD software. Therefore, a general-purpose procedure is shown in Fig. 6. The first step is to lay out the structural sketch of the Ackerman steering mechanism in several designated positions. Since the mechanism in different positions possesses the same link lengths, an equal constraint is applied to the identical link accordingly in different positions. Then, geometrical and dimensional constraints are correspondingly applied to the structural sketch such that the sketch entities possess correct degrees of freedom and perform a constrained motion of Ackerman steering mechanism. The structural sketch can be either fully constrained or under-constrained after the required constraints are applied. If the structural sketch is still under-constrained, more dimensional constraints can be placed on each link until the sketch is fully defined. The rest of the driven dimensions can be measured from the fully-defined sketch to obtain the whole specification of the synthesized Ackerman steering mechanism. A detailed operation is illustrated below.

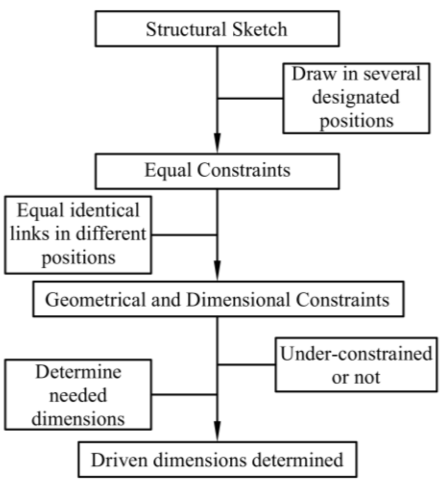


Figure 6. Geometric constraint programming

The synthesis of the configuration in the Fig. 5(a) is illustrated in Fig. 7 such that the geometry would follow Ackerman steering geometry in specified positions.

1. Locate the vehicle frame, rectangle MNOP, on the 2D sketch and impose the fixed constraints on these points, here we restrict $\overline{MN} = 100$ mm and $\overline{MO} = 200$ mm.
2. Draw a linkage skeleton A_1-F_1 for the 1st position, the middle point. Since the mechanism is symmetric, apply $\overline{A_1M} = \overline{F_1N}$, $\overline{B_1M} = \overline{E_1N}$, and $\overline{B_1C_1} = \overline{D_1E_1}$.
3. Locate the middle position for the linkage A_1-F_1. Constrain $\overline{A_1M}$, $\overline{F_1N}$, and $\overline{C_1D_1}$ to be horizontal. Also, $\overline{C_1D_1}$ is aligned with the center of vehicle frame MNOP.
4. Repeat the steps 1 and 2 for another configuration, say i^{th} displaced configuration, as illustrated in gray line.
5. Impose the equal constraints such that $\overline{A_1M} = \overline{A_iM}$, $\overline{B_1M} = \overline{B_iM}$, $\overline{B_1C_1} = \overline{B_iC_i}$, $\overline{C_1D_1} = \overline{C_iD_i}$, $\overline{D_1E_1} = \overline{D_iE_i}$, $\overline{E_1N} = \overline{E_iN}$, $\overline{F_1N} = \overline{F_iN}$, and $\angle A_1MB_1 = \angle A_iMB_i = \angle E_1NF_1 = \angle E_iNF_i$.
6. Impose the colinear constraint to $\overline{B_1C_1}$ and $\overline{B_iC_i}$ since they represent the same sliding path.
7. Impose restricted steering angle $TA_i = 45°$.
8. Set this position as the extreme position, therefore a collinearity between $\overline{E_iN}$ and $\overline{D_iE_i}$ is applied.
9. Repeat the steps 4-7 for another separated position, say j^{th} displaced configuration, which set $TA_j = 20°$.

At last, the dimensions of designed mechanism are fully determined, as listed in Table 1.

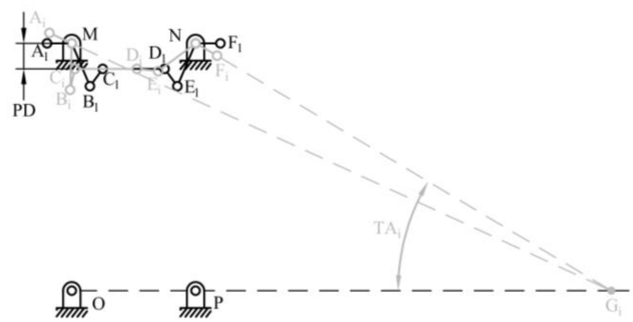


Figure 7. Six-bar linkage in its reference configuration and the i^{th} displaced configuration for Ackerman geometry

TABLE 1. DIMENSIONS OF DESIGNED MECHANISM

$\overline{MB}$	37.631 mm
$\overline{BC}$	17.394 mm
$\overline{CD}$	50 mm
$\overline{DE}$	17.394 mm
$\overline{EN}$	37.631 mm
PD	20.647 mm
$\angle AMB$	112.96°
$\angle ENF$	112.96°

V. VERIFICATION OF SYNTHESIZED MECHANISM

In this section, the synthesized mechanism with given dimensions is analyzed to verify its capability of meeting Ackerman geometry. Fig. 8 shows the error of the intersect point of two normal lines of wheel links (K_W) to the axle of the back wheel. Since the mechanism is symmetric, Fig. 8 only illustrates the results while the vehicle turns in one direction (in the right direction). According to Fig. 8, this mechanism would have visible deviations near its extreme position. In other words, this results using the GCP technology yields a smooth maneuverability of the steering mechanism for a wide range of the wheel turning.

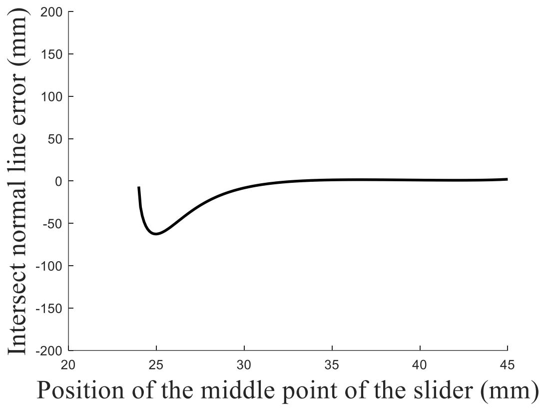


Figure 8. Variation in error of the intersect point of two normal lines of wheel links (K_W) to the axle of the back wheel

VI. Conclusion

An integrated design process which combines Yan's design methodology with the geometric constraint programming is proposed in the research. The presented process is illustrated using Ackerman steering mechanism. Two feasible linkages consisting of six links and seven joints are derived. Then, a GCP technique is applied to determine the dimensions of the linkage. Since the presented linkage has more joints and links over the conventional type using a four-bar linkage, it can meet multiple precision positions synthesis so as to obtain a smooth maneuverability of the steering mechanism for a wide range of the wheel turning, as indicated in the case study. In the future, the dimensional synthesis of the steering mechanism in Fig. 5(b) will be considered using the GCP technique as well. Also, the friction force and the bending stress of each links can be considered during the dimensional synthesis.

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